A CHARACTERIZATION OF STUDENT MATHEMATICAL THINKING THAT EMERGES DURING WHOLE-CLASS INSTRUCTION: AN EXPLORATORY STUDY

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This exploratory study investigated 164 instances of student mathematical thinking that emerged during whole-class instruction in a high-school geometry course. The MOST Analytic Framework provided a way to categorize these instances according to their building potential—that is, the potential for learning to occur if the student thinking of the instance were made the object of consideration by the class. The variations in the building potential of student thinking revealed in the study highlight the complexity of teaching and the need to support teachers in identifying and appropriately responding to instances with different levels of building potential.

Keywords: Classroom Discourse, Cognition, Instructional Activities and Practices

Reform documents (e.g., NCTM, 2014) advocate for the use of student thinking in instruction, and the benefits of doing so have been highlighted by various studies (e.g., Franke & Kazemi, 2001). Not all student thinking, however, has the same potential to support learning by being incorporated into instruction (Leatham, Peterson, Stockero, & Van Zoest, 2015), particularly if the teacher's goal is to enact the teaching practice of *building—making student thinking the "object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea"* (Van Zoest et al., 2017, p. 36). Better understanding the range of student thinking that is available during instruction could help support the development of practices that provide the most productive response to that thinking. We report here on our characterization of the student mathematical thinking that emerged during whole-class instruction in one teacher's classroom.

Literature Review

We identified research related to three types of characterizations of student thinking: (1) the context in which student thinking emerges during classroom instruction (e.g., Edwards & Marland, 1984); (2) the type of student thinking that occurs in a particular content area (e.g., Carpenter, 1985, for addition); and (3) the potential of the student thinking to further the mathematical understanding of the class if incorporated into instruction (e.g., Stockero & Van Zoest, 2013). The first two groups of studies provided insight into student thinking, but did not provide information on the potential of the student thinking to be used in instruction by a teacher. Drawing on the third group of studies, Leatham et al. (2015) developed a framework for identifying high-potential instances of student thinking that, if made the object of consideration by the class, have the potential to foster learners' understanding of important mathematical ideas.

Such instances of student mathematical thinking are called Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOSTs). Van Zoest et al. (2017) investigated the attributes of MOSTs and how those attributes might be used to support teachers' identification and productive use of MOSTs. Their study, however, did not investigate the broader range of student thinking that is available in the classroom. The exploratory study reported in this paper extends the work of Van Zoest et al. (2017) by examining the full range of student mathematical thinking that emerges during whole-class instruction and considering productive teacher responses to instances with different potential for building.

Theoretical Framework

Leatham et al. (2015) proposed six criteria for characterizing and identifying MOSTs (see Figure 1). These criteria create a natural subcategorization of student thinking that might occur in the classroom, a subcategorization that we theorized would differentiate student thinking according to the *building potential* of that thinking—that is, *the potential for learning to occur if the student thinking of the instance were made the object of consideration by the class*. We conceptualize building as a teaching practice that involves making clear the instance of student thinking that the class is to consider, turning that instance over to the class in a way that requires them to make sense of it, orchestrating the conversation that ensues, and ensuring that the mathematical point of the instance is made explicit (Van Zoest, et. al, 2016).

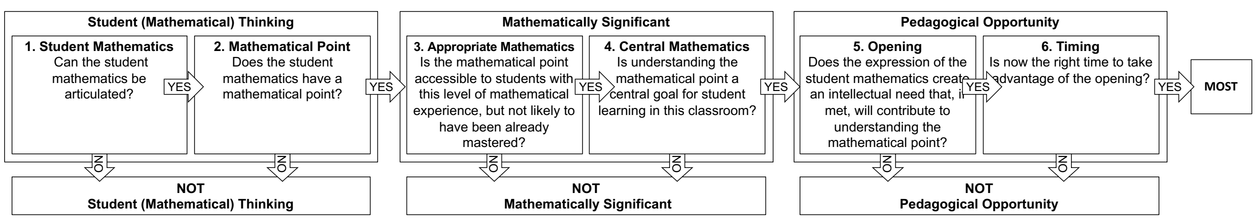

Figure 1. The Six Criteria of the MOST Analytic Framework

Methodology

This study is part of a larger project that included a data set of 11 videotaped mathematics lessons from 6-12th grade classrooms that reflected the diversity of teachers, students, mathematics, and curricula present in US schools (Van Zoest et al., 2017). Based on the analysis reported in Van Zoest et al. (2017), we identified a lesson from this data set whose rates of instances of student thinking per minute and of MOSTs per minute during whole-class instruction were at the medians of the larger data set. We analyzed the 45 minutes of whole-class instruction in this 90-minute, 11th grade geometry lesson about finding the surface area and volume of a sphere. Our unit of analysis was an instance of student thinking—an "observable student action or small collection of connected actions" (Leatham et al., 2015, p. 92)—that is potentially mathematical. Employing analytic processes described elsewhere (Van Zoest et al., 2017) resulted in 164 instances of student thinking that were coded at the last criterion they satisfied according to the MOST Analytic Framework (Figure 1). Instances that appeared mathematical, but for which the student mathematics (criterion 1) could not be inferred, were coded as Cannot Infer (CNI). We then theorized about the relationship between the location of these instances within the MOST Criteria and the potential of these instances for productively enacting the teaching practice of building. We did so by considering the potential advantages and disadvantages of enacting the building practice for such instances.

Results & Discussion

About 17% of the 164 instances were MOSTs (met all six criteria in Figure 1), and thus are, by definition, instances that are worth building on. They have *high building potential* because they are particularly opportune instances that have the potential to result in the class developing a better understanding of mathematics that is both appropriate and central to their learning. Thus, not building on a MOST would be a missed opportunity. The 34.7% of instances that met at least Central Mathematics (criterion 4) but fell short of being MOSTs, were considered to have *some building potential*. These instances are mathematically relevant, but lack pedagogical expediency—they either lack an intellectual need for the students to engage with the thinking (criterion 5) or the pedagogical timing is inappropriate to take full advantage of the thinking (criterion 6). Since these instances do involve mathematics that the students could benefit from considering, a teacher *could* use them to move the students' learning forward, but it would not be a missed opportunity were they not made an object of consideration for the class. To illustrate the difference between instances with high and some building potential, we provide an example of students engaged in sense-making for each of these categories (see Figure 2).

The teacher begins talking about great circles and a student asks a question. **Instance of Student Thinking**: Would it be kind of be like the circumference of a circle? **Mathematical Point**: A great circle is a circle of maximum circumference that can be drawn on a sphere; the great circle is the object (circle) and the circumference is the distance around the circle. **Building Potential:** High (**MOST Coding**: MOST)
The teacher asks if the surface area of a sphere is more like the area or circumference of a circle. **Instance of Student Thinking**: Area, because the circumference is just the outside, like a line [outlines a circle in the air]. And then the area's the whole thing. **Mathematical Point**: Circumference is the distance around a circle and area is the space that circle takes up. **Building Potential:** Some (**MOST Coding**: Central Mathematics)

Figure 2. Sense-Making Examples for High Building Potential and Some Building Potential

We considered the 29.9% of instances that satisfied criterion 1, 2, or 3 but fell short of meeting the criterion for Central Mathematics as having *no building potential*. These instances lack mathematical relevance to learning goals for the students in the class, so pursuing them would seem to be a pedagogical misstep. A productive teacher response to such instances might be to allow the conversation to move on. Being aware of instances that have no building potential might be as important as being aware of instances that have high building potential, since this knowledge would enable teachers to spend time where it is warranted.

The 18.9% of instances that were not inferable (CNI) were considered to have *unknown building potential*. Pursuing these instances as the object of whole-class discussion in their current form would also seem to be a pedagogical misstep. With these instances, if students attempt to infer their peers' mathematical ideas, they may arrive at very different interpretations. Participating in a discussion with different perceptions of what it is that they are discussing could lead to miscommunication and inhibit learning (Peterson et al., 2018). Although often these instances do not seem to warrant further consideration because of their limited nature, in some cases the students' idea might very well be relevant and timely to the discussion (possibly even a MOST). In these cases, a productive teacher move would be to seek clarification.

Conclusion

The results of this exploratory study showed that student thinking available in a high school geometry classroom during whole-class instruction varied substantially, both with regard to the MOST Criteria and with regard to its building potential. We identified and discussed four categories of building potential (high, some, none, and unknown), each of which suggests different types of productive responses from a teacher. The variation in building potential of instances of student thinking revealed in this study highlights the complexity of teaching, and the need to support teachers in identifying and appropriately responding to instances with different levels of building potential. Further work is needed to better understand these various response-related practices and how they might be coordinated into productive use of student mathematical thinking. Knowing more about variations in building potential and associated productive responses informs the design of professional development that supports teachers in implementing instruction that productively incorporates student mathematical thinking.

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