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Attributes of Student Mathematical Thinking that is Worth Building on in Whole Class Discussion

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Motivation

- Benefits of instructional practices that use student thinking
  - Fennema, et al., 1996 [CGI]
  - Stein & Lane, 1996 [QUASAR]
  - NCTM, 2014
- Not all student thinking is the same
  - MOSTS: Leatham, Peterson, Stockero & Van Zoest, 2015

Mathematically significant pedagogical **O**pportunities to build **i**n **S**tudent **T**hinking

MOST
Analytic Framework

Stockero, S. L., Peterson, B. E., Leatham, K. R., & Van Zoest, L. R. (2014). The "MOST" productive student mathematical thinking. *Mathematics Teacher,* 108(4), 308-312.

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education,* 46(1), 88-124.

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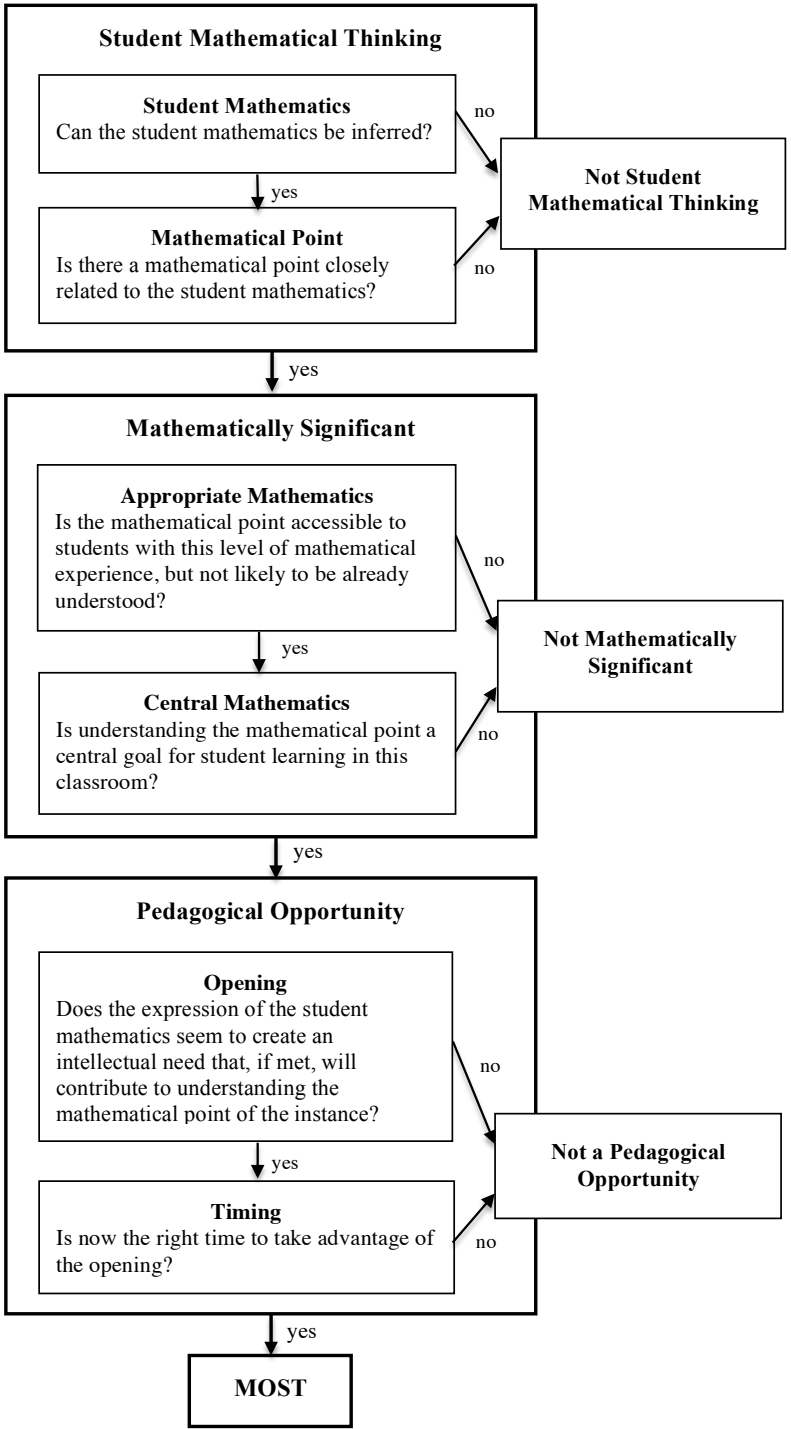

Mathematical Opportunities in Student Thinking

MOSTs are opportunities…

…for the teacher to make student thinking the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

…to **build** on student thinking.

Motivation

- Benefits of instructional practices that use student thinking
  - Fennema, et al., 1996 [CGI]
  - Stein & Lane, 1996 [QUASAR]
  - NCTM, 2014
- Not all student thinking is the same
  - MOSTs: Leatham, Peterson, Stockero & Van Zoest, 2015
- Mathematical noticing is difficult
  - Jacobs, Lamb & Philipp, 2010
  - Sherin, Jacobs & Philipp, 2011
  - Stockero, Rupnow & Pascoe, 2015
- Better understanding attributes of useful student thinking might help

Methods

- 11 mathematics lessons
- 10 teachers
- California, Hawaii, Michigan, Mississippi, New Mexico & Utah
- Whole-class interactions (349 minutes)
- Unit of analysis was an instance of student thinking

MOST Analysis

Mathematical Opportunities in Student Thinking

- Pass 1 - Identified each instance of student mathematical thinking
  - 1571 instances in the 349 minutes of whole-class interaction
  - 4.5 instances per minute (range of 1.2 to 5.8 across 11 lessons)

- Pass 2 - Used the MOST Analytic Framework (Leatham et al., 2015) to identify MOSTs
  - 278 of the 1571 instances of student thinking were MOSTs
  - 0.8 MOSTs per minute (range of 0.1 to 1.5 across 11 lessons)

Research Question

What are the attributes of MOSTs that secondary school mathematics teachers have available during instruction?

MOST Analysis

- Pass 1 - Identified each instance of student mathematical thinking
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- Pass 2 - Used the MOST Analytic Framework (Leatham et al., 2015) to identify MOSTs
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- Pass 3 – Investigated attributes of MOSTs (coding scheme expanded from Stockero & Van Zoest, 2013)
  - Seven attributes
  - Fall into two groups

Coding for Attributes

Mathematical Opportunities in Student Thinking

Locus (Where does the MOST fall in the terrain?)

Attribute	Definition	Codes
Prompt	The invitation or lack thereof that precipitated the MOST	• Spontaneous • Open Invitation Spontaneous • Open Invitation Selected • Targeted Invitation
Basis	Whether the student mathematics (SM) in the MOST is based on earlier work or in-the-moment thinking	• In-the-Moment • Pre-Thought
Math Goal	The distance of the mathematical idea of the MOST from the day's lesson goals	• Lesson • Unit • Course • Discipline

Coding for Attributes

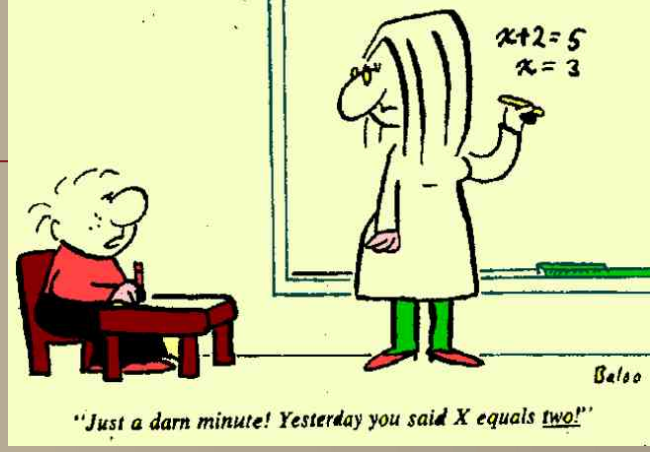
Mathematical Opportunities in Student Thinking

Cognition (How is the student thinking expressed?)

Attribute	Definition	Codes
Form	The way in which the student thinking is expressed	• Question • Tentative Statement • Declarative Statement
Accuracy	The validity of the student mathematics (SM) of the MOST	• Correct • Incorrect • Incomplete • Combination • N/A
Intellectual Need	The extent to which the compelling nature of the MOST is transparent	• Obvious • Translucent • Hidden
Type	The nature of the SM that made the instance compelling	• Incorrect or Incomplete • Sense Making • Multiple Ideas or Solutions • Information

"Just a darn minute! Yesterday you said X equals <u>two!</u>"

MOST Attributes: Example



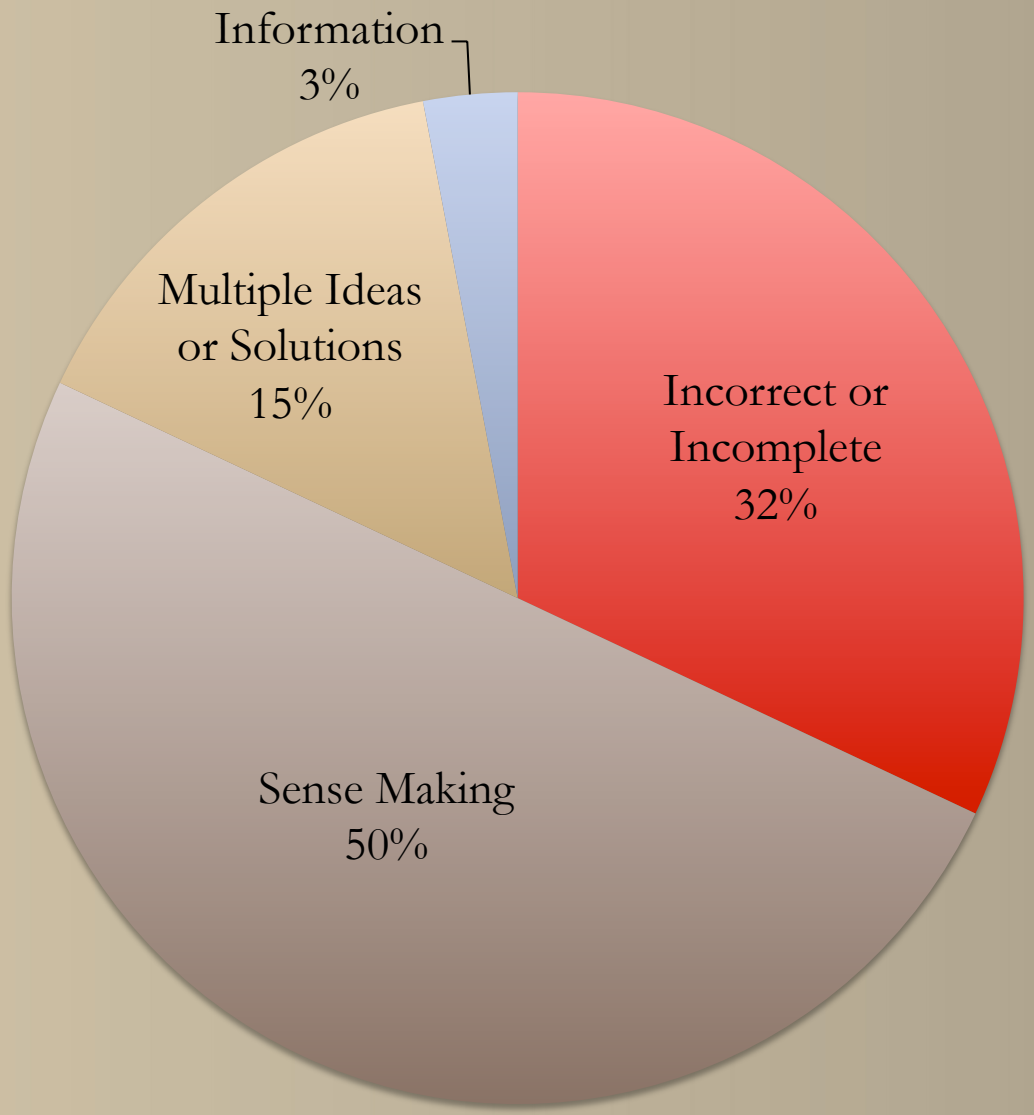
Student Mathematics: Yesterday x equaled 2 and today x equals 3.

Mathematical Point: A letter can be used to represent an unknown quantity in an equation and can represent different values for different equations.

Prompt: Spontaneous
Basis: In-the-Moment
Math Goal: Course

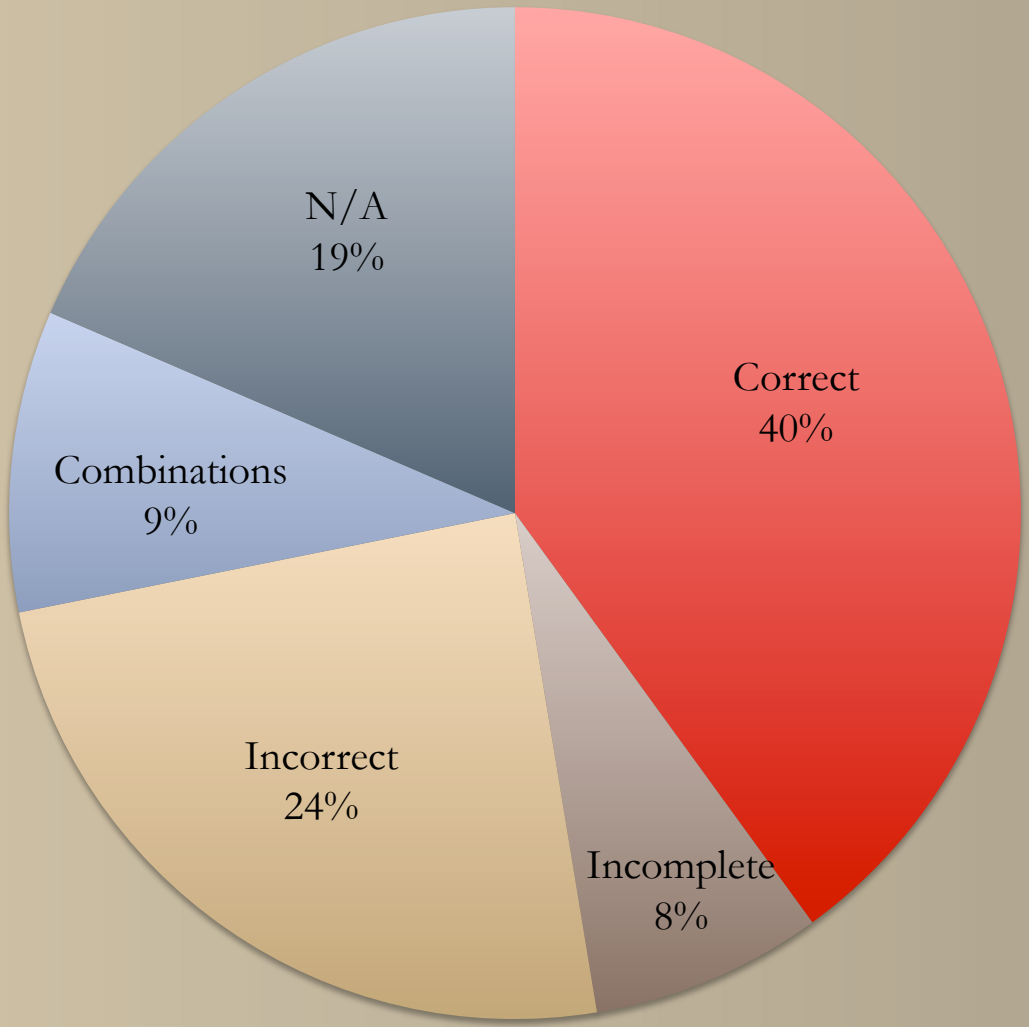
Form: Declarative
Accuracy: N/A
Intellectual Need: Obvious
Type: Sense Making

MOST Type

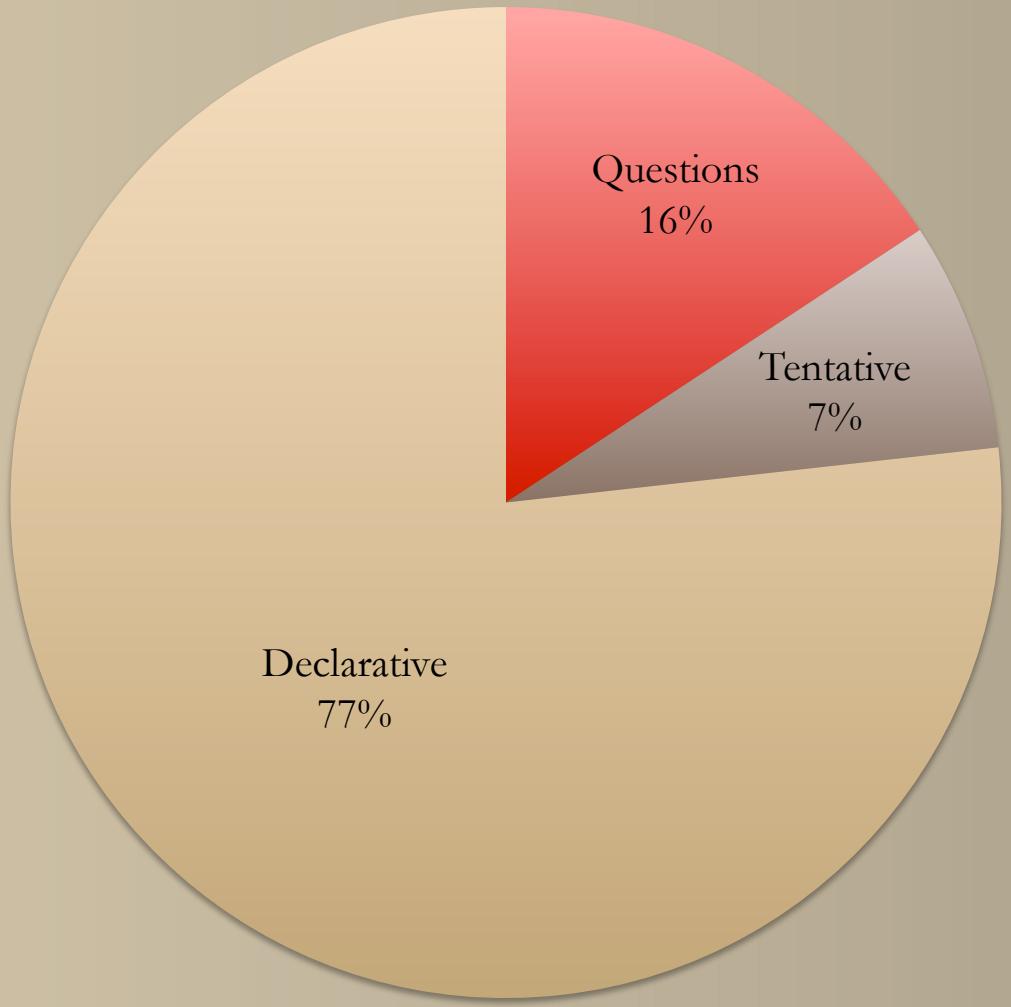

Information
3%
Multiple Ideas
or Solutions
15%
Incorrect or
Incomplete
32%
Sense Making
50%

MOST Accuracy

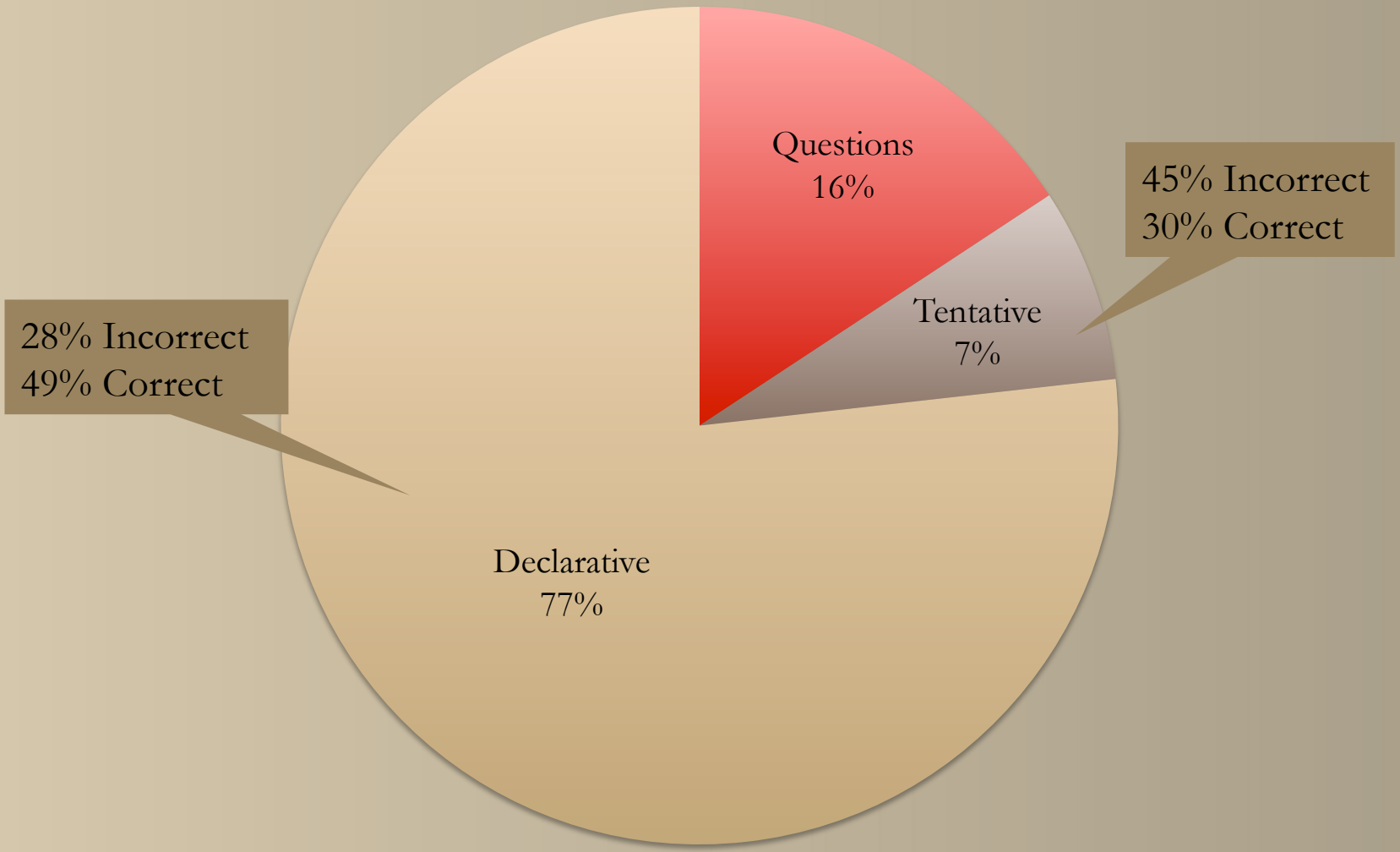
Mathematical Opportunities in Student Thinking



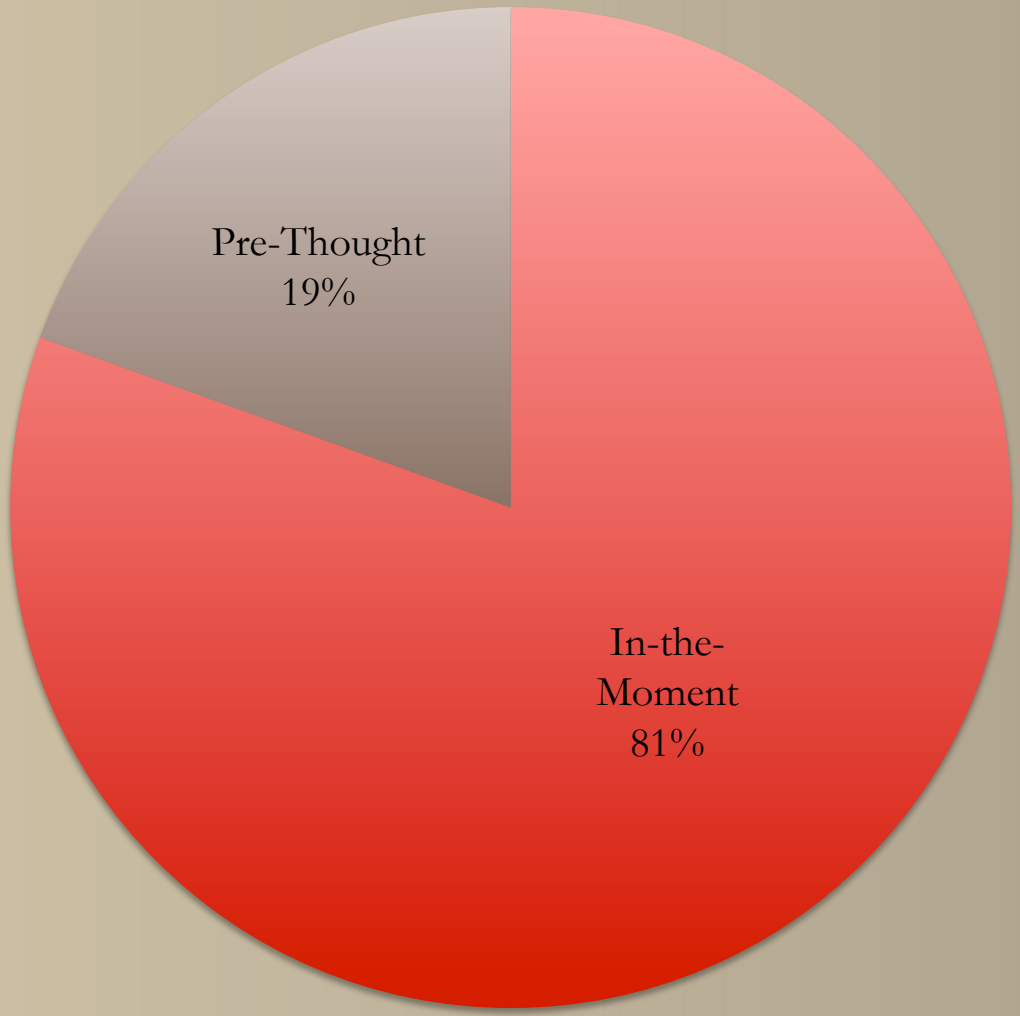
MOST Forms



MOST Forms

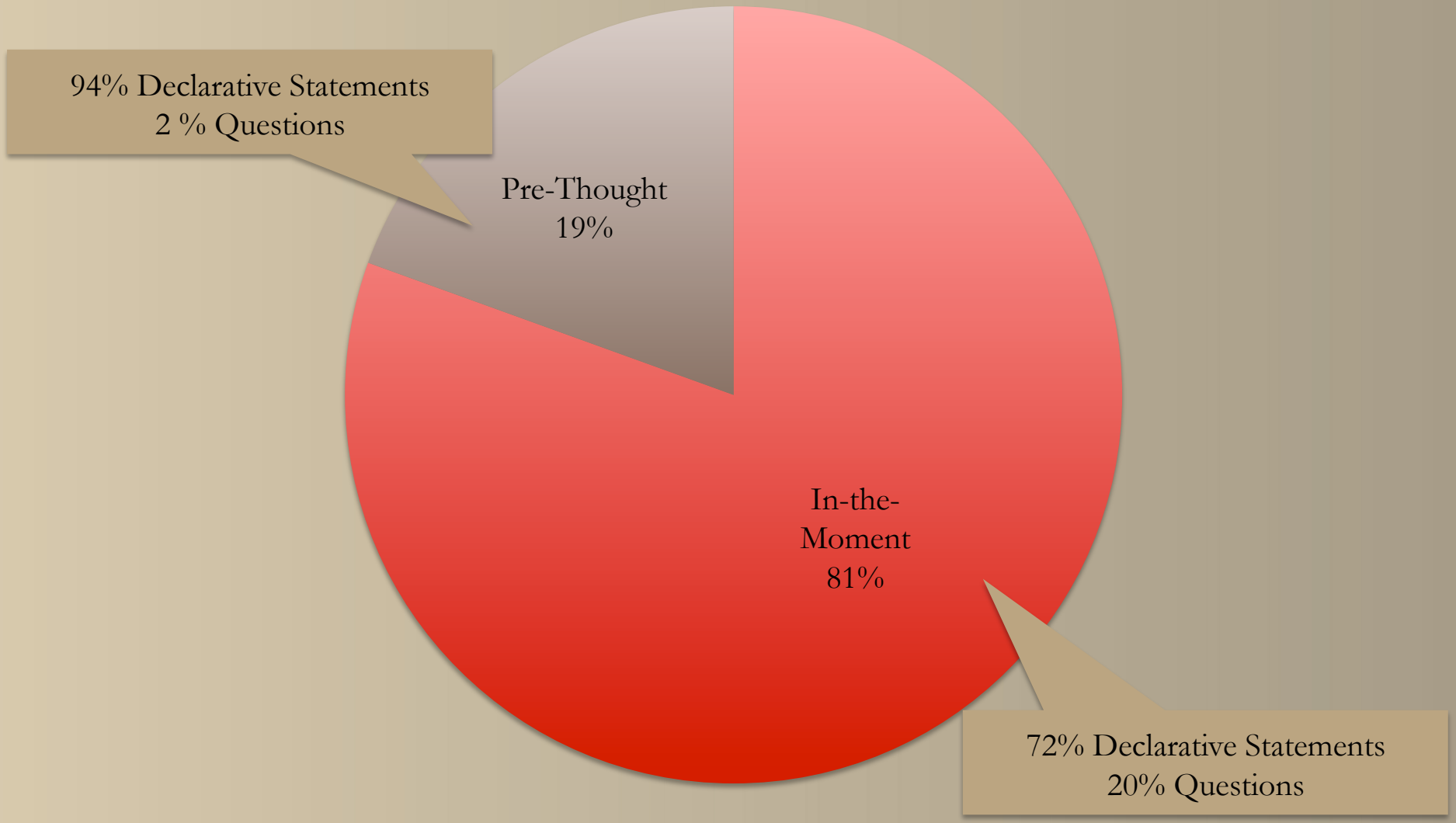

Questions
16%
45% Incorrect
30% Correct
Tentative
7%
28% Incorrect
49% Correct
Declarative
77%

MOST Basis



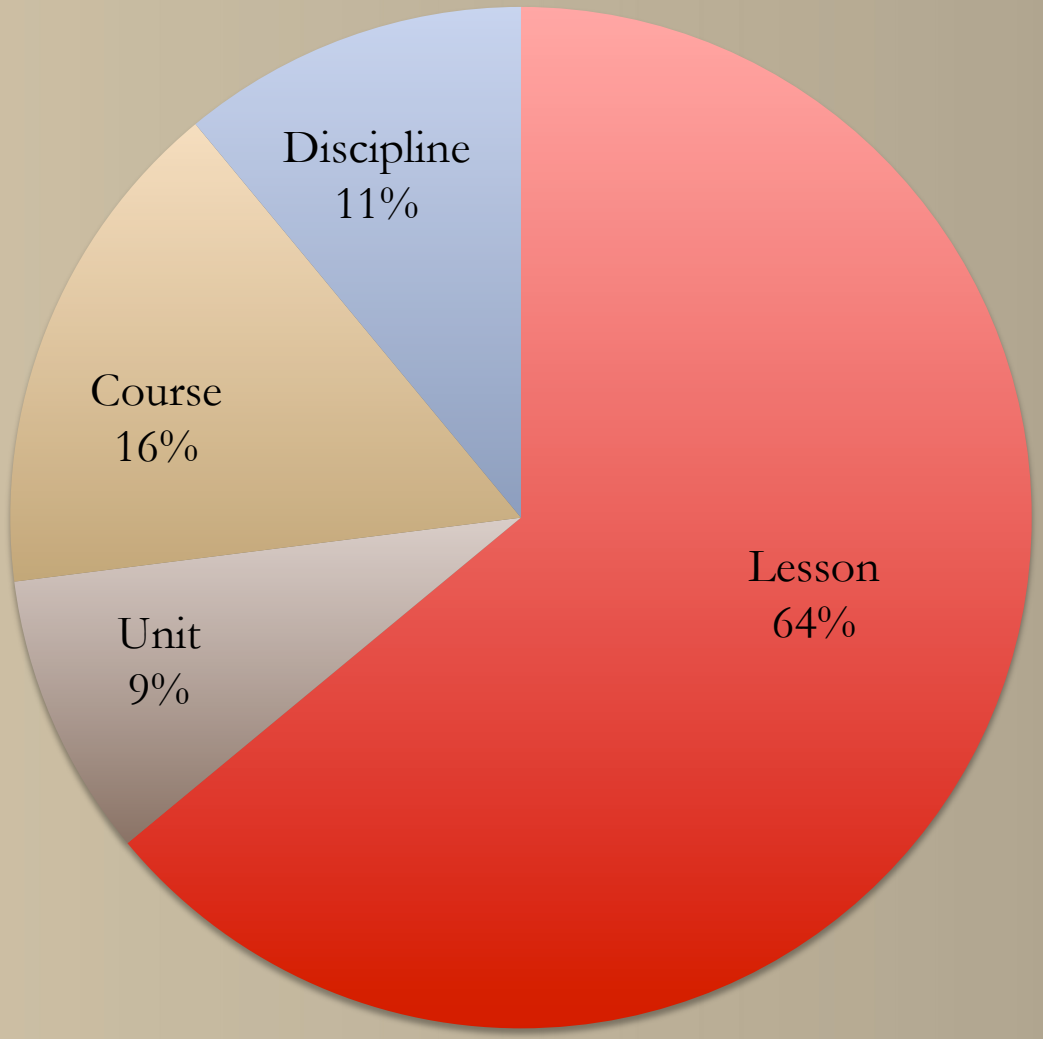
MOST Basis

Mathematical Opportunities in Student Thinking



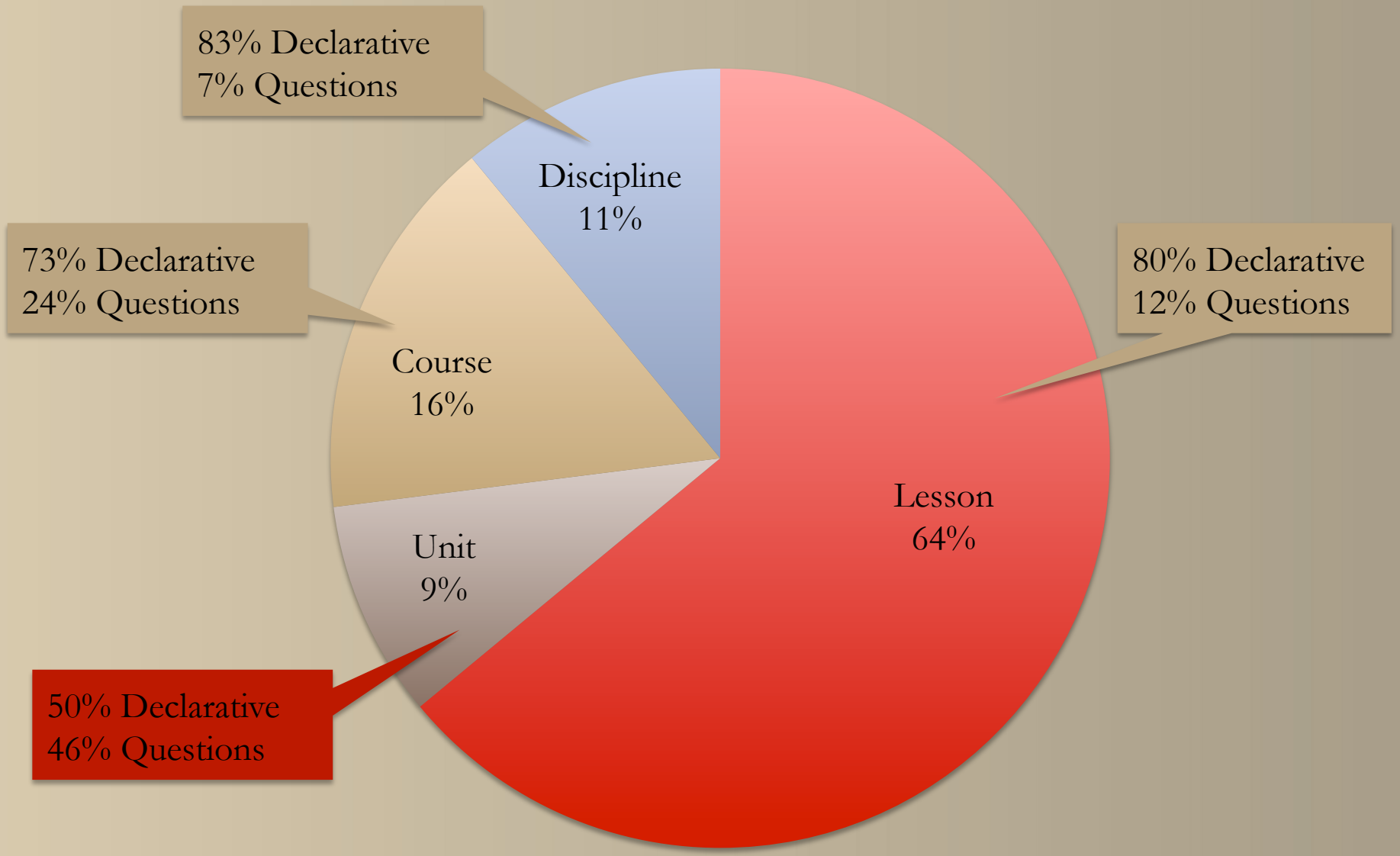
MOST Mathematical Goal

Mathematical Opportunities in Student Thinking

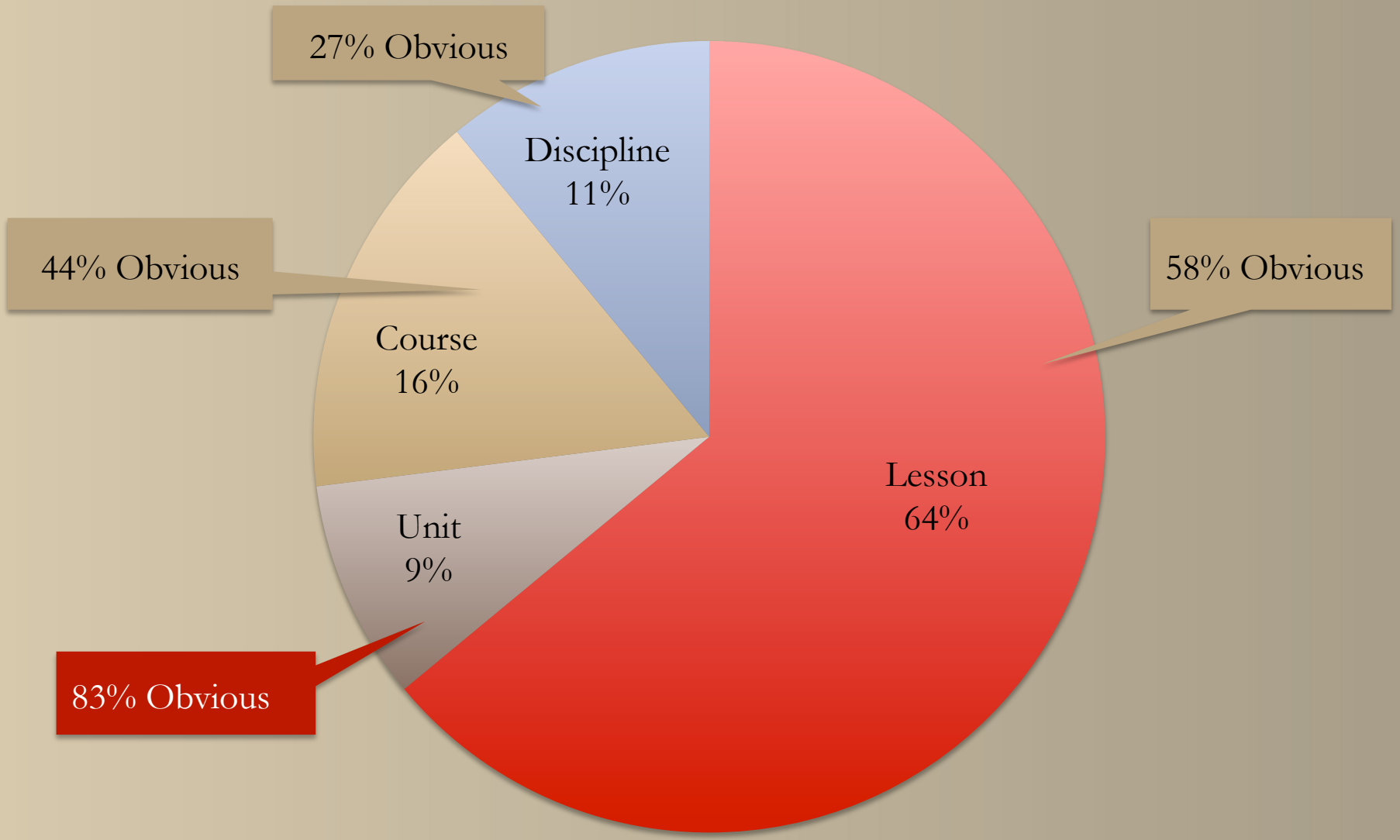


MOST Mathematical Goal

Mathematical Opportunities in Student Thinking



MOST Mathematical Goal



Implications

- Characterizing Student Mathematical Thinking
  - MOSTs fall into a small number of type categories
  - Important to attend to all forms of thinking that surface during instruction
  - Connections to broader goals for student learning
- Supporting Student Thinking Centered Instruction
  - Complexity of student thinking that teachers have available
  - Nuances of student thinking
- Development of Assessments for Teaching

Questions?

For more information, see

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