THE COMPLEXITY OF INTERPRETING STUDENT THINKING AND INFERRING ITS POTENTIAL TO FOSTER LEARNING

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Research has shown that listening to and interpreting student thinking is challenging, yet critical for effective incorporation of student mathematical thinking (SMT) into instruction. We examine an exemplary teacher's interpretations of SMT, his inference of the potential of the SMT to foster learning, and the rationale for his responses to that thinking. Our findings reveal some reasons why teachers may fail to successfully act on SMT that emerges during whole class discussion. This study confirms previous research findings, that in order to incorporate SMT into instruction in a way that fosters learning, teachers must correctly interpret that SMT. The study also shows that even good teachers may need support in developing skills that will enable them accurately interpret SMT and its potential to foster learning.

The use of student mathematical thinking (SMT) to inform instructional decisions has been emphasized in current calls for reform as captured in US National Council of Teachers of Mathematics documents (e.g., NCTM, 2014). Using SMT in ways that can foster learning requires making sense of that thinking (Maher & Davies, 1990). However, listening to student thinking is challenging (Ball, 1993) and sometimes teachers incorrectly interpret that thinking (Maher & Davies, 1990). Understanding teachers' perceptions of SMT that emerges during instruction and their reasons for using that thinking in particular ways would enable teacher educators to be responsive to teachers in efforts to support them in enhancing their teaching practice around the use of SMT. To that end, the purpose of this exploratory study is to understand an exemplary teacher's interpretations of SMT and rationale for using the student mathematics available to him during whole class discussion in the way that he did.

LITERATURE REVIEW

Research has shown the instructional value of teachers making sense of SMT. The research of the Cognitively Guided Instruction (CGI) project has shown that providing teachers with opportunities to make sense of SMT has a positive impact on teacher learning and student achievement (Franke & Kazemi, 2001). Therefore, it is important that researchers and teacher educators understand teachers' processes of making sense of SMT so as to identify the kinds of support teachers may need in order to develop the skill of accurately inferring SMT. Studies that have examined teachers' interpretations of SMT include those that have examined a broad swath of classroom activity, of which SMT is only a part (e.g. van Es & Sherin, 2002), and those that have

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focused specifically on SMT (e.g. Crespo, 2000; Jacobs, Lamb & Philip, 2010). In this brief literature review we consider studies that have focused specifically on SMT.

Research has shown that interpretation of SMT is a skill teachers can develop. Jacobs, Lamb & Philip (2010) examined teachers' development of three skills—*attending* to children's strategies, *interpreting* children's understandings, and *responding* to those understandings—and found that these three skills can be learned. Crespo (2000) studied Preservice Teachers (PSTs) interpretations of SMT by examining how PSTs interpreted SMT as they exchanged letters with 4th grade students. The results showed that PSTs' interpretations changed from being evaluative to making sense of students' mathematical solutions and from making quick conclusions about student abilities to making more thoughtful interpretations. Crespo's (2000) study also highlighted the importance of attending to conversations that reflect teachers' processes of interpretation. Our work in this paper is informed by this finding on the importance of attending to such conversations. In this exploratory study we analyze conversations between the researcher and the teacher to infer the teacher's interpretations of SMT and their perception of the potential in the SMT to foster learning. Other studies that have provided further insight into teachers' development of the skill of interpreting SMT include van Es and Sherin (2008), which revealed that teachers follow different pathways as they develop the skill of interpreting SMT.

Some studies on teachers' interpretations of SMT have revealed factors that may influence teachers' inferences of SMT. Maher and Davies (1990) revealed that a teacher's limited understanding of a mathematical concept can impede the correct interpretation of SMT. Other factors that may influence a teacher's interpretation of SMT include the teacher's orientations towards listening to students. Davis (1996) described three such orientations: evaluative, interpretive and hermeneutic. Teachers with an evaluative orientation listen to SMT in order to determine the correctness of the SMT. Those with an interpretive orientation listen to SMT in order to get the student's understanding while those with a hermeneutic orientation listen to SMT by engaging with students in a process of negotiation of meaning and understanding. An understanding of the factors that influence teachers' interpretation of SMT could inform the design of professional development that would support teachers to accurately infer SMT and incorporate it in instruction in ways that foster learning.

Incorporating SMT in instruction in ways that foster learning while at the same time optimizing limited instructional time requires that incorporation of SMT be informed by the potential in the SMT to foster learning. Many studies that have examined teachers' interpretation of SMT have not explicitly addressed the potential of the SMT to foster learning, yet not all SMT has the same potential to foster learning (Leatham et al., 2015). Identifying the potential in SMT to foster learning is critical if teachers are to productively use SMT. In this study we explore a teacher's interpretation of SMT that occurred in his classroom with a view to better understand how he interprets SMT and his perception of the potential in that thinking to foster learning. We therefore seek to answer the following research questions: 1) How does an exemplary teacher interpret SMT that emerges in-the-moment during whole class instruction? 2) What inferences does an exemplary teacher make about the potential of SMT, that emerges in-the-moment during whole class instruction, to foster learning of mathematical ideas?

THEORETICAL FRAMEWORK

Our examination of a teacher's interpretations of SMT and his perceptions of the potential of that thinking to support learning of important mathematical ideas is guided by the MOST Analytic Framework, where a MOST is defined as a "Mathematically Significant Pedagogical Opportunity to Build on Student Thinking" (Leatham et al., 2015, p. 90). We refer to the practice of taking advantage of MOSTs as *building* (Van Zoest et al., 2016) and define building as making an instance of SMT "the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea" (Van Zoest et al., 2017, p. 36).

MOSTs occur at the intersection of three critical characteristics of classroom instances: student mathematics (SM), significant mathematics, and pedagogical opportunity. For each characteristic, two criteria determine whether an instance of SM embodies that characteristic. For SM the criteria are: "(a) one can observe student action that provides sufficient evidence to make reasonable inferences about *SM* and (b) one can articulate a mathematical idea that is closely related to the SM of the instance—a *mathematical point (MP)*" (Leatham et al., 2015, p. 92). The criteria for significant mathematics are: "(a) the MP is *appropriate* for the mathematical development level of the students and (b) the MP is *central* to mathematical goals for their learning" (p. 96). Finally, "an instance embodies a pedagogical opportunity when it meets two key criteria: (a) the SM of the instance creates an *opening* to build on that thinking toward the MP of the instance and (b) the *timing* is right to take advantage of the opening at the moment the thinking surfaces during the lesson" (p. 99). The six *MOST Criteria* are considered linearly and an instance of SMT is classified according to the last criterion it satisfies (SM, MP, Appropriate, Central, Opening, and Timing). Those instances that appear mathematical, but for which the SM cannot be inferred, are designated *cannot infer* (CNI). When an instance satisfies all six criteria, it is a MOST.

METHODOLOGY

This study is part of a larger project focused on understanding teachers' in-the-moment responses to SMT during whole class instruction (see LeveragingMOSTs.org). The teacher who is the focus of this exploratory study was chosen because he regularly incorporates student thinking into his lessons and has been recognized by his school district and university mathematics educators as an exemplary teacher. The data for this study consisted of four videotaped math lessons from this teacher's classroom and seven corresponding follow-up interviews.

Our data analysis focused on four different units of analysis: instances of SMT, the teacher's in-the-moment responses to those instances, the teacher's retrospective interpretation of the SMT, and the teacher's retrospective reasoning for his responses. An instance of SMT consists of "an observable student action or a small collection of connected actions" (Leatham et al., 2015, p. 92). The teacher's in-the-moment response to an instance consists of the teacher actions that begin "as a given instance of SMT ends and ends when that instance of SMT is no longer the focus of the observable teacher actions" (Peterson et al., 2017, p. 19). The teacher's retrospective interpretation of the SMT consists of interview instances in which the teacher explains his interpretation of an instance of SMT from the classroom video. The teacher's retrospective reasoning for his response to an instance of SMT consists of interview instances in which the teacher explains what he was thinking when he made a specific pedagogical move in class. We used video analysis software to segment each classroom lesson into the instances of SMT and the teacher responses to each individual instance. We used the same software to divide each interview into segments where instances of SMT from the classroom videos were discussed. Only those instances that the interviewer thought were likely to be MOSTs, appeared to be treated as MOSTs, or instances that the teacher wished to discuss, were addressed in the interviews.

We began coding by identifying instances of SMT that were specifically discussed in the interviews. These instances of SMT were located on the video recorded classroom lessons and were coded using the MOST Analytic Framework (Leatham et al., 2015), and the teacher responses to these instances of SMT were coded using the Teacher Response Coding (TRC) framework (see Peterson et al., 2017). For our third and fourth units of analysis, we categorized the teacher's responses in the interviews as either focusing on the students' thinking or the teacher's response. We used open coding to further categorize the instances dealing with SMT, and the instances dealing with the teacher's responses.

The goal of this study is to understand teachers' in-the-moment responses to SMT. Therefore, the teacher's interpretations of SMT relative to the MOST Analytic Framework are examined simultaneously with the teacher's reflections on his responses to that thinking. We examine the teacher's reflections on his responses to SMT and how those reflective thoughts align with the potential of that thinking to foster learning of important mathematical ideas.

RESULTS AND DISCUSSION

Using open coding on the teacher's interview responses focusing on students' thinking, three categories emerged: the teacher's inferences of a) the SM, b) the MP, and c) the instance's placement within the MOST Analytic Framework. Using open coding on the teacher's interview responses focusing on his in-the-moment responses to students' thinking, two categories emerged: the teacher's rationale for his responses, and his reflection on those responses.

From the four lessons and seven interviews we identified 34 instances of SMT where the teacher gave some insight into his understanding of, or reaction to the SMT. Thus, in addition to knowing what the student said and the teacher's in-the-moment reaction, we were also able to discern one or more of the following: how the teacher interpreted what the student said, how the teacher placed the SMT within the MOST framework, the teacher's rationale for his in-the-moment response, or the teacher's reflection on his in-the-moment response. Through analyzing these factors, we saw that even in a classroom with an excellent teacher, barriers that may prevent building still exist.

Our analysis revealed that the majority of the time–18 out of 34 instances–the teacher's determination of the building potential either aligned with the MOST Analytic Framework, or we had no indication from the data that his determination differed from the framework. When the teacher recognized the building potential of SMT in an instance, his placement of these instances on the MOST framework were in line with the MOST Analytic Framework. Additionally, the teacher often explicitly spoke of the building potential that he saw in instances, and how this guided his in-the-moment responses to these instances. When both the teacher and the MOST Analytic Framework categorized instances as MOSTs, the teacher reported that his in-the-moment responses to SMT were meant to harness that building potential, which was corroborated by both the teacher's retrospective reflection on his actions as well as our coding of the teacher's response to that SMT.

For 14 of the 34 instances, the difference in building potential between the MOST Analytic Framework, and the teacher's explicit or implicit response can be grouped into four categories: a) taking up only part of the SMT because the teacher did not understand all of it, b) taking up only part of the SMT because the teacher focused on some aspect of that part of the SMT, rather than all of the SMT, c) the teacher considered additional context or thinking that was not part of the SMT because he wanted the conversation to move in a specific direction, a direction that the SMT alone would not likely have steered it, and d) the teacher did not see the importance in an instance of SMT. The first three of these were the most common; there was only one instance of d), so we only focus on the first three. Finally, of the 34 instances, there were two instances where the teacher made an inference of the SMT, which according to the MOST Analytic Framework, could not be inferred. This could have been due to classroom norms, or the teacher's insights to his students, but it was not clear to us that there was a shared understanding of what the student said by the rest of the class.

To help exemplify the three most common types of instances where the teachers did not recognize the building potential in SMT, we will look first at an instance where the teacher did not take up all of the SMT because he did not understand all of it. In one such instance, the class was working with geometric sequences, and in trying to understand the formula for a geometric sequence a student said: "every time we plug in a number it gives us, like, one term further than we wanted it to. So, if we subtract one from n then it puts us back one term every time." In-the-moment, the teacher did not fully understand what the student was saying, and the teacher interpreted the SMT as dealing with plugging values in to check the validity of an equation. After the teacher watched this portion of the class on video, he realized that the student was actually working towards understanding that altering the exponent in the formula of a geometric sequence will yield a different term in the sequence, which is something that could have been built upon to increase the understanding of the whole class. Thus, the teacher was clearly able to see that a building opportunity existed when he had time to contemplate the SMT, but unfortunately, he was not able to see this in the moment, and a building opportunity was missed.

Next, we look at an instance where the teacher only took up part of the SMT because he focused on some aspect of that part of the SMT, rather than all of the SMT. In this example, the class was beginning to work with logarithmic equations, and a student said that "If you add $\log_2 2$, $\log_2 9$, and $\log_2 2$ you get the same thing as $\log_2(9 \cdot 2 \cdot 2)$.", and went on to note that there was a relationship between this idea and the exponential product rule. The teacher then focused only on the exponential product rule and asked the class "when you multiply exponents you add them together. What does that mean?" and after a student responded to this, the teacher dropped this instance of SMT, and moved on. Since the students in this class were already familiar with the exponential product rule, the teacher missed an opportunity to connect students' prior knowledge to the new material that they were currently grappling with. This differs from our last example because–as he explained in the interview–the teacher understood what the student was saying, but in the moment, he made the conscious decision to take up only a small portion of the SMT that was presented. In retrospect, the teacher was able to see that this was indeed a missed opportunity, and he noted that he should have further utilized this instance of SMT.

Finally, the third most common category of instances where the teacher missed an opportunity to build on SMT, were instances where the teacher considered additional context or thinking that was not part of the SMT because he wanted the conversation to move in a specific direction, a direction that the SMT alone would not likely have steered it. In one such instance, the class was working with geometric sequences and a student said, "Okay, um, we got $a_n = 2(4)^n$." Earlier in the lesson, other students had noticed that each term in the sequence that they were given was four times the previous term. Drawing on this earlier thinking and this newly presented equation, the teacher inferred that the SM was "you begin with 2 and continually multiply by 4." Unfortunately, by drawing on this earlier thinking, the teacher missed the opportunity to build on the thinking at hand, which was that a student had, for the first time in this class, presented an equation for a geometric sequence. Since the class had been trying to use their knowledge of exponents to help them construct the formula of a geometric sequence, the fact that after grappling with it, one student had finally presented such a formula would have been the perfect opportunity to build on that SMT to help the rest of the class understand how to derive this formula. However, since the SMT also lent itself to a point that another student had been trying to make earlier, the teacher decided instead to use the current SMT to exemplify an earlier student response.

CONCLUSION

This study confirms the results found by Maher and Davis (1990) that for teachers to productively use SMT it is important that they accurately infer that thinking. Our findings show that incorporating SMT in ways that foster learning requires that in addition to correctly interpreting SMT teachers need to also correctly identify the potential of the SMT. This will help deepen or enrich the mathematical understanding of the other students in the class by making the SMT the object of consideration. Furthermore, the complexity of teaching is highlighted in this study as we find that even an exemplary teacher may not always be able to utilize appropriately the in-the-moment SMT that is available to them. As our study showed, possible reasons for this include: not completely understanding the SMT, choosing deliberately not to act on a portion of the SMT, or adding additional context to the SMT to make a specific MP. Regardless of the exact reason, not fully utilizing available SMT can lead to missed opportunities for learning when the instances are MOSTs–"Mathematically Significant Pedagogical Opportunity to Build on Student Thinking" (Leatham et al., 2015, p. 90). Our findings show that in two out of 34 instances the teacher inferred the SMT where according to the MOST Analytic Framework, SMT could not be inferred. Such inferences could result in unproductive engagement in situations where the class does not have a shared understanding of the SMT when the SMT is ambiguous and the teacher response does not seek clarification of the instance before allowing the class to engage with it. This study confirms findings of Cobb (1988) and Maher and Davis (1990), that it is important that teachers carefully listen to and seek to fully understand, and clarify if necessary, the SMT that occurs in their classrooms since not doing so can hinder students' construction of mathematical ideas as the teacher is then more likely to impose their own constructions on students. This suggests that there may be a need for professional development that is focused on supporting teachers to make sense of SMT that emerges in-the-moment during their instruction as a first step towards the development of the skill of building on SMT. Future studies could identify ways of supporting teachers' development of skills that would facilitate the accurate interpretation of SMT and its underlying potential to foster learning.

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